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Theoretical prediction of the sizes of drops formed in the breakup of capillary jets

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Abstract – The breaking up of a liquid jet subject to small disturbances was studied experimentally and theoretically. A non-linear theory was used to calculate the profile of the waves on the surface of the jet at breakup, and also to predict the volume of the main and satellite drops.

It was found that the theoretical and experimental drop volumes were in good agreement. However, satellites were observed in all the experiments whereas the theory predicts that satellites should not form for dimensionless wavenumbers greater than 0·7.

In the experiments it was not possible to make the jets break up into single main drops of uniform size. The results suggest that it may be possible to produce uniformly sized drops if the main and satellite drops have the same volume.

INTRODUCTION

IN MANY chemical engineering operations the production of liquid drops is an important step in providing a large surface area for heat or mass transfer. Although the complete process of spraying or atomization from the initial release of the liquid to the final dispersion of the drops is a very complex chain of events, there are a number of cases where the breakup of a jet of liquid or small ligament is the most important step in determining the drop size. Thus a detailed study of liquid jet instability is an obvious prerequisite if any advance is to be made in the understanding of the overall phenomenon.

Particular examples of processes involving simply the breakup of a jet due to surface tension are those in which some fusible product is required in the form of spherules. The practice is to melt the solid and let it run out of a vessel with a perforated bottom, at the top of a tall tower. The liquid jets formed as the liquid runs out of the holes break up into droplets which then solidify as they fall. Lead shot has been made in this way for centuries; a modern-day example of products made in this way is in various forms of fertilizer. Of particular interest in these industries is the need to make droplets of uniform size. If a distribution of drop sizes is generated the small ones are overtaken by the larger ones and coalesce while solidifying into undesirable agglomerates. Also, cooling air is often blown up the towers so that very small particles may be entrained and lost.

Because of its technological importance, a great deal of work has been done on spraying and atomization. (Comprehensive reviews may be found in the work of Brodkey[1] and Orr[2].) However most of these have been experimental. Although the foundations had been laid by Rayleigh in the last century, it is only recently that an improvement has been made on his linearized theory.

The capillary instability of liquid jets was studied theoretically by Rayleigh for inviscid and very viscous jets. The results of the theoretical study and early experiments are given by Rayleigh[3]. These experiments and earlier ones by Plateau and Savart showed that when a jet broke up into drops there was not simply one drop per wavelength, but that very often, especially at long wavelengths, smaller intermediate drops were produced. These spherules

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or "satellites" were assumed to have been produced by the growth of some harmonic of the original disturbance.

Rayleigh's theoretical analysis was a linearized one, that is the products of velocities and their derivatives were neglected and the form of the perturbation was assumed to be a sinusoid and to remain sinusoidal during the growth of the instability. Thus the volume of the jet was only conserved to the first order in the perturbation amplitude and the production of small intermediate droplets is not predicted.

Apart from the production of satellite droplets, other non-linearities in the shape of the wave have been observed by Crane, Birch and McCormack[4], in the form of "bunches" of liquid which occur along the jet at the positions where the wave crest should occur. These liquid plates were the result of applying perturbations of very large amplitude to the jet at the orifice from which it formed. Even in cases where an initial perturbation of small amplitude has been used, such as in the work of Donnelly and Glaberson[5], non-sinusoidal waves have been observed near the point of breakup of the jet. Thus although the linear theory will predict the behaviour of the jet for small fluid motions, it cannot describe the shape of the jet near the point of breakup and hence cannot correctly predict the size distribution which will result from an unstable jet. In order completely to characterize the behaviour of an unstable jet, the theory must include terms which allow the perturbed surface to assume a shape other than sinusoidal and a condition must be imposed on the solution to ensure that the volume of the jet will be constant for terms higher than the first order of the perturbation amplitude. Such an analysis has recently been given by Yuen[6], and a particularly interesting result is that swellings or secondary waves are formed between the crests of the primary disturbance waves. These secondary waves are not harmonics of the primary wavelength, but are shown to be the result of energy transfer from the fundamental and lower order harmonics into those of higher order.

In this paper we use the theory of Yuen to calculate the wave profile at the time when the deepest trough in the surface coincides with the jet axis, when breakup is assumed to occur. It is therefore possible to predict the volume of liquid in the "main" drops (from the crests of the primary disturbance) and the "satellite" drops (the remainder). These predictions are shown to be in excellent agreement with experimental data.

THEORETICAL

To determine the shape of the surface of the jet for small time but starting with a finite perturbation, Yuen[6] assumed that the surface disturbance could be expressed as a series expansion:

$$\eta(z, t) = \sum_{m=1}^{\infty} \eta_0{}^m \eta_m \tag{1}$$

where $\eta(z, t)$ is the equation of the surface and η_0 is the initial perturbation. The amplitude of the perturbation (ϵ_0) is such that

$$\eta_0 = 1 + \epsilon_0 \tag{2}$$

and ϵ_0, η_0 are dimensionless with respect to the jet radius r_j. The coefficients in the first order approximation were chosen to make the solution agree with that obtained in the analysis of Rayleigh[3]. The second order solution contained a purely time dependent term to offset the addition of volume from the fundamental. The solution was extended to third order terms, resulting in

$$\begin{aligned}\eta = {} & \eta_0 \cos Kz \cosh \omega_1 t \\ & + \eta_0{}^2 (B_{22}(t) \cos 2Kz - \tfrac{1}{8}(\cosh \omega_1 t + 1)) \\ & + \eta_0{}^3 (B_{31}(t) \cos Kz + B_{33}(t) \cos 3Kz)\end{aligned} \tag{3}$$

where z is the axial distance, K the wavenumber, both dimensionless with respect to r_j. The time t is dimensionless with respect to $(r_j^3 \rho/T)^{1/2}$. The coefficients are given by Yuen[6]. Typographical omissions were found in the expressions for b_{22}, c_{22}, which should read:

$$b_{22} = \frac{2\omega_1{}^2 I_b(1 - 2KI_a) + \{2 + K^2 + \omega_1{}^2(3 - I_a{}^2)\}\, K}{4I_b(\omega_2{}^2 - 4\omega_1{}^2)} \tag{4}$$

$$c_{22} = \frac{2+K^2+\omega_1{}^2(1+I_a{}^2)}{8(1-4K^2)} \qquad (5)$$

where I_a, I_b are ratios of Bessel functions given by Yuen.

Using Eq. (3), wave profiles were calculated for values of time needed for the trough to coincide with the centreline, it being assumed that breakup would then occur. Examples are shown in Figs. 1, 2 and 3, together with the dimensionless breakup time. For wavenumbers less than 0·7, satellite droplets are predicted while above 0·7 there is no intermediate swelling between nodes of the fundamental frequency, hence no satellites are predicted. For $K = 0{\cdot}5$, an anomolous result occurs, probably due to the high emphasis which the theory places on the second order terms.

On the assumption that all the liquid enclosed by the primary wave crest formed the "main" drop, and that the liquid under the secondary wave formed a "satellite" drop, the drop sizes at jet breakup were calculated for initial perturbations of 1 and 10 per cent of the jet radius. The

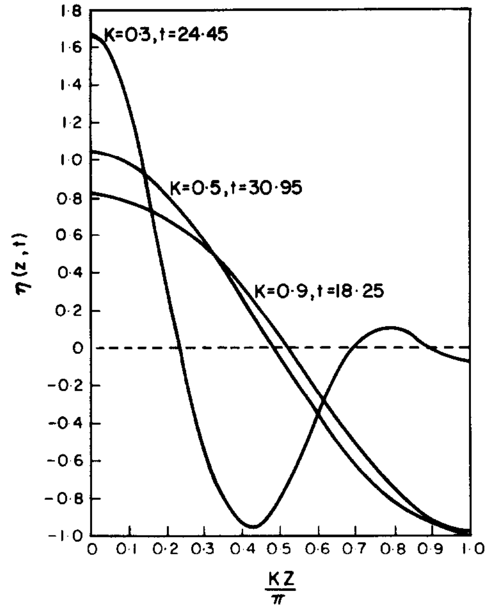

Fig. 2. Wave profiles at breakup for dimensionless wave numbers $K = 0{\cdot}3, 0{\cdot}5$ and $0{\cdot}9$ ($\epsilon_0 = 0{\cdot}01$).

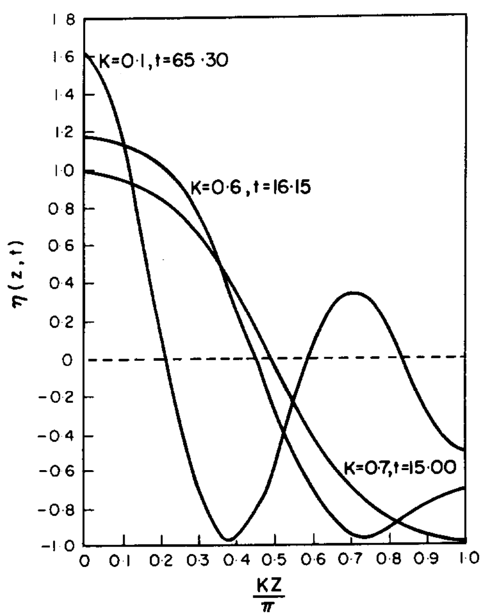

Fig. 1. Wave profiles at breakup for dimensionless wavenumbers $K = 0{\cdot}1, 0{\cdot}6$ and $0{\cdot}7$ ($\epsilon_0 = 0{\cdot}01$).

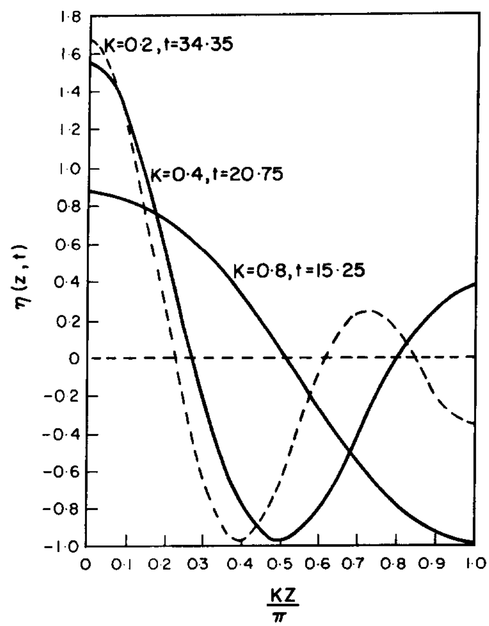

Fig. 3. Wave profiles at breakup for dimensionless wave numbers $K = 0{\cdot}2, 0{\cdot}4$ and $0{\cdot}8$ ($\epsilon_0 = 0{\cdot}01$).

results are shown in Table 1 in terms of r_d/r_j where r_d is the equivalent radius of the drop. Because the theory is not strictly valid for large times, volume is not necessarily conserved as the wave amplitude increases. Thus at breakup on the above assumption, the total volume of the liquid enclosed by the calculated waveform is not equal to the volume of the original jet. The error is given in Table 1 expressed in the form of an equivalent radius, so that if r_p is the equivalent radius of the volume enclosed by the wave profile at breakup,

$$\text{error} = 100(r_p - r_j)/r_j\,\%. \quad (6)$$

Table 1. Drop sizes predicted by non-linear theory

K	r_d/r_j, $\epsilon_0 = 0{\cdot}01$: Main	Satellite	Error (Eq. 6),	$\epsilon_0 = 0{\cdot}1$: Main	Satellite	Error (Eq. 6)
0·05	3·70	4·38	12·8	3·83	4·28	12·6
0·10	2·89	3·47	11·8	3·25	3·51	18·0
0·15	2·66	3·03	14·2	2·80	3·10	18·1
0·20	2·37	2·63	10·6	2·74	2·60	17·5
0·25	2·09	2·48	9·0	2·11	2·51	10·3
0·30	2·08	2·42	12·9	2·09	2·51	17·0
0·35	2·07	2·17	12·3	2·09	1·93	6·5
0·40	2·08	1·61	3·9	2·07	1·19	−3·3
0·45	2·05	1·40	3·1	2·04	0·86	−4·2
0·50	2·42	0·008	14·6	2·41	0	14·6
0·55	1·99	0·76	−1·0	1·99	0·69	−1·9
0·60	2·00	0·44	1·5	2·00	0·40	0·9
0·65	2·00	0·17	3·5	1·99	0·17	3·1
0·70	1·95	0·02	3·6	1·95	0	3·2
0·75	1·91	0	3·9	1·90	0	3·3
0·80	1·90	0	4·9	1·88	0	4·3
0·85	1·88	0	6·1	1·87	0	5·5
0·90	1·87	0	7·9	1·86	0	7·5
0·95	1·87	0	9·9	1·86	0	9·1
1·00	1·90	0	13·8	1·88	0	12·2

The error is greatest for long waves where the time to breakup is longest, and least in the region of maximum instability. Since it is generally not possible to specify the amplitude of the initial disturbance in practice, it is interesting to note that the values of r_d/r_j do not change markedly with a tenfold increase in ϵ_0.

EXPERIMENTAL

A schematic diagram of the apparatus is shown in Fig. 4. Jets of water were produced from a straight delivery tube of inside diameter 4 mm, and length 30 cm. The breakup length of the undisturbed water jets was at least 60 cm. The jet velocity past the measuring station was in the range 250–350 cm/sec. This is sufficiently high to make the effect of acceleration due to gravity small (less than 10 per cent change in velocity over the jet length, assuming free fall) and yet not high enough to cause appreciable aerodynamic effects.

To avoid introducing disturbances into the liquid stream from running machinery, and to provide a constant liquid flowrate for a period of time, an air pressure pumping system was used. By maintaining the pressure over the liquid in the pressure tank above 40 psig, the pressure change due to a fall in liquid level in the tank during a run of 20 min duration was less than 2

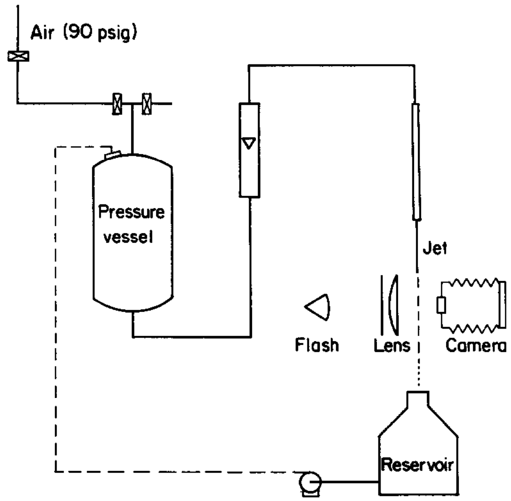

Fig. 4. Schematic diagram of the apparatus.

per cent. The volume of the tank was approximately 30 l. All the piping was $\frac{3}{8}$ in. copper tube.

A rotameter was used to set up successive runs with approximately constant flowrates. The actual flow was measured by catching the liquid from the jet in a graduated cylinder over a known interval of time.

In order to study the jet breakup in detail it is necessary to be able to perturb the jet at a controlled and known frequency. The perturbation must be of such a magnitude that the perturbed jet breaks up in a length shorter than the unperturbed breakup length, and yet if one is to study breakup as it occurs from the growth of an infinitesimal disturbance so that some approach to the linearized theory can be made in the experiments, the disturbance must be small enough so that no wave is visible for some distance downstream of the perturbation. Donnelly and Glaberson[5] showed the feasibility of using a loud-speaker near the jet or a fine needle immersed in the jet to provide a small perturbation. Direct disturbance of the liquid in this way avoids any damping which might result from the mechanical transmission of natural frequencies if some portion of the apparatus from which the jet is flowing is vibrated.

Small amplitude vibrations were produced from a 6-in. 10 W speaker driven by a Radford 15 W power amplifier. The perturbation frequency was generated by a signal generator (model J-1, Advance Components Ltd.,) with a frequency range from 15 to 50,000 c/sec. The amplitude of the perturbation was controlled by varying the power output of the signal generator and was always such that the jet travelled about 20 cm before any visible wave formed, but broke up in a length shorter than the breakup length of the undisturbed jet. The jet could be viewed in the light of a stroboscope, so that experiments could be set up and studied qualitatively before a photograph was taken.

To observe the jet instability in as much detail as possible a photographic system was used which would provide large plates with a minimum of distortion. A Schneider–Kreuznach Repro-Claron 135 mm, f8 lens was used on a MPP sliding back 4 × 5 plate camera. The image length produced on the plate was approximately 0·6 times full size, so that it was possible to photograph up to 22 cm of jet length. The light source was a Dawe Microflash Junior which gives 8·5 J with about 10 μsec. duration. The light was diffused near the source, further diffused after travelling 50 cm and then passed through a 25 cm dia. condensing lens to provide a background of uniform illumination. Exposures were made on Ilford FP4 glass plates at f22.

Measurements of the wave profiles and drop sizes were taken from the plates using a Hilger Universal Measuring Projector. The plate was enlarged X15 on to a ground glass screen. It was possible to read the coordinates to ±0·00005 in., and the error in positioning the crosswires on the wave crest was ± 0·001 in. in the axial direction and ± 0·0001 in. in dia. The unperturbed jet radius was measured for each run. All measurements of wavelength and radius were made in the top half of the plate where possible so that acceleration over the measured length would not be appreciable. The wavelengths used in calculating the results are in each case the average of a number of waves along the jet, so that errors in determining the position of the wave crest would tend to cancel. Full details of the experimental and measuring techniques are given in Rutland [12].

RESULTS

Figure 5 shows a series of water jets perturbed at frequencies corresponding to wavenumbers from 0·075 to 0·683. In all these cases a secondary non-linear wave can be seen, generally symmetrically placed.

Figure 5(a) shows instability at a very long wavelength. The bottom droplet is produced from a secondary wave, whereas the other large drop is from the main wave crest. The instability of the ligament is also clearly seen. In Fig. 5(b), the first and third large drops from the bottom are main drops while the second is from a secondary wave. The amplitude of the main wave at the top of the jet is greater than the secondary wave with its crest opposite the 13 cm mark on the scale. The instability of the free ligament results in a number of small drops.

At the shorter wavelength in Fig. 5(c) the secondary wave is less pronounced, and occurs as a swelling in the ligament just before the main drop. The wave is not symmetrically placed between the two primary crests, but is displaced toward the downstream drop. The effect is clearly seen in the shape of the first free ligament, and affects the ligament breakup, as seen in the second and third ligaments where large drops form at the lower ends.

Very regular secondary waves are seen in Fig. 5(d). The second and fourth drops from the bottom come from the main wave crest, and the first and third are from the secondary wave. The main drops are slightly larger than the secondary drops. The behaviour of the ligament is again irregular but since the ligaments are shorter than in the previous illustrations, in most cases only one drop is formed from them. This point is emphasized in Fig. 5(e) where the satellite is very small and in some cases coalesces with the main drop.

The results of the theoretical and experimental analysis of the drop sizes produced from the water jets are shown in Fig. 6 and Table 3. The lines were drawn using the theoretical points shown in Table 1, smoothed to eliminate the volume error for each wavenumber and initial perturbation. The smooth data are shown in Table 2. As was remarked earlier, the size of the main drops does not appear to be sensitive to the amplitude of the initial perturbation ϵ_0. Accordingly each of the theoretical curves in Fig. 6 is the mean of the two values for a given wavelength presented in Table 2.

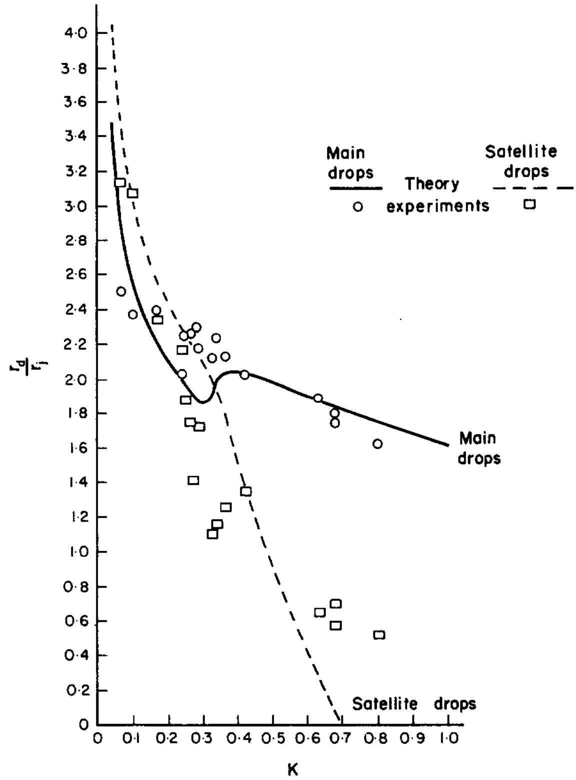

Fig. 6. Comparison of predicted and measured drop sizes from the break up of water jets.

In some cases, especially at long wavelengths, the ligaments broke up into further satellite drops. For representation purposes, all of the material not in a main drop is taken as forming a single satellite drop.

The agreement between the experiments and the predictions is good at high and low wavelengths. In the range of K from 0·25 to 0·4 there is a considerable scatter in the experimental results. It is in this region that the theoretical

Table 2. Smoothed drop size data from non-linear theory

K	$\epsilon_0 = 0{\cdot}01$ $(\frac{r_d}{r_j})$ main	$\epsilon_0 = 0{\cdot}01$ $(\frac{r_d}{r_j})$ satellite	$\epsilon_0 = 0{\cdot}1$ $(\frac{r_d}{r_j})$ main	$\epsilon_0 = 0{\cdot}1$ $(\frac{r_d}{r_j})$ satellite
0·05	3·28	3·89	3·40	3·80
0·10	2·59	3·11	2·76	2·98
0·15	2·33	2·66	2·37	2·63
0·20	2·15	2·39	2·33	2·21
0·25	1·92	2·28	1·94	2·30
0·30	1·83	2·12	1·79	2·15
0·35	1·85	1·93	1·96	1·81
0·40	2·00	1·55	2·15	1·23
0·45	1·99	1·36	2·13	0·90
0·50	2·11	0·007	2·12	0
0·55	2·00	0·77	2·02	0·70
0·60	1·98	0·44	1·98	0·39
0·65	1·94	0·17	1·94	0·16
0·70	1·89	0·015	1·89	0
0·75	1·84	0	1·84	0
0·80	1·81	0	1·81	0
0·85	1·77	0	1·77	0
0·90	1·73	0	1·73	0
0·95	1·71	0	1·71	0
1·00	1·68	0	1·68	0

Table 3. Drop sizes from the disintegration of water jets

K	f (c/sec)	λ (cm)	r_j (cm)	Q (cc/sec)	$\frac{r_d}{r_j}$ (main)	$\frac{r_d}{r_j}$ (satellite)
0·075	40	8·50	0·1014	10·2	2·51	3·14
0·110	50	6·19	0·1081	10·6	2·38	3·08
0·174	80	3·80	0·1054	10·8	2·40	2·36
0·250	85	2·89	0·1151	11·0	2·04	2·18
0·258	97	2·26	0·0927	5·9	2·26	1·89
0·272	100	2·38	0·1030	9·4	2·27	1·76
0·281	150	2·23	0·0998	10·2	2·29	1·42
0·297	150	2·23	0·1055	11·3	2·18	1·74
0·336	150	1·95	0·1046	10·2	2·13	1·13
0·348	150	2·03	0·1124	11·0	2·24	1·17
0·371	180	1·73	0·1018	10·2	2·14	1·27
0·430	200	1·62	0·1107	11·8	2·03	1·35
0·641	300	1·03	0·1051	10·8	1·90	0·66
0·683	300	0·79	0·0859	5·0	1·80	0·71
0·683	320	0·96	0·1046	9·6	1·75	0·58
0·806	340	0·72	0·0928	6·1	1·63	0·53

main drop radius has a local minimum, and a transition occurs with the theory predicting that the main drops are larger than the satellites for $K > 0{\cdot}35$. This trend is followed by the experimental results.

The theory does not predict satellite production beyond the point of maximum instability, $K = 0{\cdot}697$. However in the experiments, satellites, albeit small, were obtained in this region. Donnelly and Glaberson[5] also report satellites up to $K = 0{\cdot}678$ at least. Experimenters using smaller jet radii found that the wavenumber above which single sized drops were formed was 0·45[7, 8], but they used extremely small jets and did not actually photograph or measure the individual drops produced. Their assertions that uniformly sized drops were obtained were based on visual observations. On the other hand a careful investigation of droplet production was carried out by Park and Crosby[9]. They photographed and measured drop sizes and found that it was possible to make drops without satellites for disturbance wavenumbers greater than 0·35. Two possible explanations offer themselves. First there may be some scaling effect such that with very small jets the ligaments between nodes break at one end only, and remain attached to one of the main drops. This would certainly give a monosize dispersion.

An alternative argument, expecially in view of the above results, is that satellites do form—but they are of the same volume as the main drops. It is particularly suggestive that all of the above investigators found that the optimum wavenumber for production of uniform drops was less than that of the most unstable wave, $K = 0{\cdot}697$. Our theoretical and experimental results showed that in the range of K between 0·3 and 0·5 the main and satellite drops were approximately of the same size.

The non-linear theory of Yuen is for jets of inviscid liquid, i.e., cases where the surface tension predominates over the viscous stresses. The appropriate dimensionless group is $J = Tr_j/\rho\nu^2$, so that the theory is essentially for $J \to \infty$. In order to test the effect of viscosity on satellite production, a number of experiments were carried out with glycerol solutions giving J as low as 0·34 compared with J of order 10^5 for the water jets. The results are shown in Table 4. In comparison with the data in Fig. 6 it is seen that at high viscosities the satellites become very small and the main drops would approach the size predicted by the linear theory. The growth rates of disturbances on these liquids are, of course, very low and since the growth of the secondary waves is a second order process in that they are fed by energy from the lower harmonics, viscosity has a greater effect on them than on the primary disturbance. For the water jets it was found that satellites were smallest near the most unstable wavenumber, because here the time to break up is a minimum. At this wavenumber the waves are also quite short, another factor reducing the length and volume of the satellites. With viscous jets however, the most unstable wavenumber depends on the parameter J, as can be seen in Fig. 7, which was calculated by the full linearized theory[10] and also by the simplified theory of Weber[11]. For J of order 1·0 the most unstable wavelength is double that for water. Thus with increasing viscosity the damping of the secondary waves is to some extent offset by an increase in their length.

CONCLUSIONS

The non-linear theory, when used to calculate the shape of the surface of a capillary jet at the point of breakup, appears accurately to predict the volume of the main and satellite drops. For long disturbance waves the satellites are larger

Table 4. Drop sizes from disintegration of viscous jets

K	f (c/sec)	Q (cc/sec)	λ (cm)	r_j (cm)	T (dyn/cm)	ρ (g/cm^3)	μ (P)	J	$\frac{r_d}{r_j}$ (main)	$\frac{r_d}{r_j}$ (satellite)
0·117	50	17·0	6·88	0·1283	32·1	1·195	0·59	14·0	3·37	0·84
0·144	60	17·0	5·62	0·1283	32·1	1·195	0·59	14·0	3·14	1·22
0·154	130	2·7	2·24	0·0549	64·2	1·244	3·25	0·42	3·09	0·18
0·169	130	3·8	2·37	0·0637	64·0	1·249	3·90	0·34	3·06	0·22
0·193	80	17·0	4·17	0·1283	32·1	1·195	0·59	14·0	2·84	1·09
0·241	100	17·0	3·34	0·1283	32·1	1·195	0·59	14·0	2·66	0·94
0·283	103	5·5	2·12	0·0952	40·0	1·115	0·05	1,700	2·16	2·11
0·485	140	11·2	1·68	0·1295	56·0	1·120	0·04	4,600	1·99	1·21

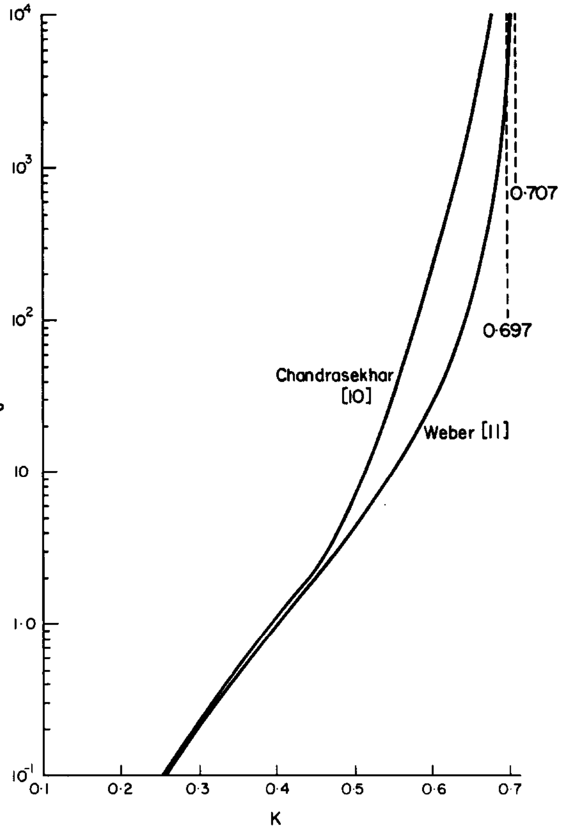

Fig. 7. The wavenumber of the fastest growing disturbance as a function of the parameter $J = Tr_j/\rho\nu^2$.

in total volume than the main drop, while for $K > 0{\cdot}35$ the theory predicts that the main drop is larger than the satellite. This is confirmed by the experiments. It is not possible to describe the behaviour of the ligaments once breakup has occurred, but qualitatively, the longer the wavelength, the greater is the number of drops. For $K > 0{\cdot}4$, only two drops were generally seen per wavelength.

The theory predicts that for $K > 0{\cdot}7$ there are no satellites formed. However we found that satellites were always formed, although they were very small at large K.

Although our results show that satellites are always formed, it may be possible to make a jet break up into droplets of uniform size by introducing disturbances of wavenumber between 0·35 and 0·50. Here the main and satellite drops are of approximately the same size.

The effect of increasing viscosity is to produce smaller satellite drops. The ligaments are more stable, and hence the total number of droplets formed per wavelength is generally fewer than in the corresponding case for water.

NOTATION

- J $Tr_j/\rho\nu^2$
- K wavenumber, $2\pi/\lambda$, dimensionless
- r_d equivalent radius of drop
- r_j radius of jet
- r_p equivalent radius of volume of liquid beneath $\eta(z, t)$ over one wavelength
- t time, dimensionless
- T surface tension
- z axial distance, dimensionless

Greek symbols

- ϵ_0 amplitude of initial perturbation, dimensionless
- η equation of surface of jet
- η_0 initial perturbation ($\eta_0 = 1 + \epsilon_0$)
- λ wavelength
- ν kinematic viscosity
- ρ density

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Résumé—La rupture d'un liquide soumis à de petites perturbations a été étudiée expérimentalement et théoriquement. Une théorie non-linéaire a été employée pour calculer la courbe des ondes sur la surface d'un jet au point de rupture et aussi pour prédire le volume des gouttes principales et satellites.

Les volumes des gouttes tirés de la théorie et de l'expérience sont en bon accord. Toutefois, des satellites ont été observés dans toutes les expériences alors que la théorie prédisait que les satellites ne pouvaient se former pour des nombres d'ondes sans dimensions supérieurs à 0,7.

Il n'a pas été possible, dans les expériences courantes, de rompre les jets en gouttes principales séparées de même grandeur. Les résultats suggèrent qu'il serait possible de produire des gouttes de même grandeur si les gouttes principales et satellites ont le même volume.

Zusammenfassung—Der Zerfall eines Flüssigkeitsstrahls infolge geringfügiger Störungen wurde experimentell und theoretisch untersucht. Es wurde eine nicht-lineare Theorie verwendet zur Berechnung des Wellenprofils an der Oberfläche des Strahls beim Zerfall, und auch zur Vorhersage des Volumenes der Haupt- und Nebentropfen.

Es konnte festgestellt werden, dass die theoretischen und experimentellen Tropfenvolumen gut übereinstimmten. Es wurden jedoch in allen Versuchen Nebentropfen beobachtet, während laut Theorie die Bildung von Nebentropfen für dimensionslose Wellenzahlen von über 0,7 ausbleiben sollten.

In den Versuchen war es nicht möglich die Strahlen zum Zerfall in einzelne Haupttropfen gleichför miger Grösse zu bringen. Die Ergebnisse deuten darauf hin, dass es möglich sein könnte, Tropfen gleichförmiger Grösse zu erzeugen, wenn die Haupt- und Nebentropfen das gleiche Volume aufweisen.